

shock surface can then be written as

$$[\rho_\alpha(\mathbf{U} - \mathbf{v}_\alpha) \cdot \mathbf{N}] + \hat{\epsilon}_\alpha = 0 \quad (10)$$

In this equation,  $\mathbf{N}$  denotes the unit normal to the shock surface and  $\hat{\epsilon}_\alpha$  is the surface mass supply for  $S_\alpha$  on the shock surface. Except for the term  $\hat{\epsilon}_\alpha$ ,

Equation (10) is identical to the usual mass jump equation for a single material. The extra term allows for mass transfer among the mixture constituents. It accounts for the growth (or loss) of  $S_\alpha$  mass within the shock surface.

The balance of momentum jump equation for  $S_\alpha$  is given by

$$[\rho_\alpha \mathbf{v}_\alpha \otimes (\mathbf{U} - \mathbf{v}_\alpha) + \mathbf{T}_\alpha] \cdot \mathbf{N} + \hat{\mathbf{m}}_\alpha = 0 \quad (11)$$

Here  $\hat{\mathbf{m}}_\alpha$  is called the surface momentum supply for  $S_\alpha$ . It accounts for the growth or decay of  $S_\alpha$  momentum within the shock surface. Finally, the energy jump equation for the constituent  $S_\alpha$  is given by

$$\left[ \rho_\alpha \left( \epsilon_\alpha + \frac{1}{2} v_\alpha^2 \right) (\mathbf{U} - \mathbf{v}_\alpha) + \mathbf{T}_\alpha \mathbf{v}_\alpha + \mathbf{h}_\alpha \right] \cdot \mathbf{N} + \hat{\epsilon}_\alpha = 0 \quad (12)$$

where  $\hat{\epsilon}_\alpha$  is the surface energy supply for  $S_\alpha$  within the shock surface. Like the mass and momentum supplies previously defined,  $\hat{\epsilon}_\alpha$  allows for energy exchange among the mixture constituents within the shock.

Equations (10), (11), and (12) are correct balance statements crossing any surface within the mixture, shock or not. If we are to use them effectively below, however, we must assume the existence of a stable shock front passing through the mixture. This being done, we point out that the jump equations apply only to the limit values of the various quantities  $\rho_\alpha$ ,  $\mathbf{v}_\alpha$ , etc. leading and following the shock. No assumption of a perfect step pulse is made, nor is it considered appropriate for many mixtures.

Following the usual program of mixture theory, we sum Equations (10), (11), and (12) over the range  $\alpha = 1, 2, \dots, k$  to obtain the mixture jump equations. These equations express conservation of mass, momentum, and energy in the whole mixture. We employ Equations (1) and (3) through (9) as well as the postulate that the mean motion of the mixture must obey the jump conditions for a single material. The mass, momentum, and energy equations for the mixture become respectively

$$\sum_\alpha \hat{\epsilon}_\alpha \equiv 0 \quad (13)$$

$$\sum_\alpha \hat{\mathbf{m}}_\alpha \equiv \mathbf{0} \quad (14)$$

$$\sum_{\alpha} \hat{\epsilon}_{\sigma} \equiv 0 \quad (15)$$

For a more detailed derivation of these three relations, see Reference [1].

Next we wish to make a distinction between the physical configuration of the composite material and the idealized mixture configuration outlined above. The mixture equations are based on the concept that each constituent is itself a continuum, and any place in the mixture may be simultaneously occupied by all of the constituents. It is unlikely that this will be true of the physical configuration of the composite, where generally the constituents will remain distinct from one another. The two configurations, physical and mixture, should not be confused. Rather we desire to employ both in order to best represent the composite material. To accomplish this we introduce the volume fraction  $n_{\alpha}$  for the constituent  $S_{\alpha}$ . We define  $n_{\alpha}$  as the ratio of the volume of  $S_{\alpha}$  in the physical configuration per unit volume of the mixture. Clearly then

$$\sum_{\alpha} n_{\alpha} \equiv 1 \quad (16)$$

We view  $n_{\alpha}$  on a macroscopic scale only, determining it on an appropriately sized neighborhood of the point in question.

We can now define certain quantities in the physical configuration. The crystal density,  $\tilde{\rho}_{\alpha}$ , for the constituent  $\alpha$  is defined by

$$\rho_{\alpha} = n_{\alpha} \tilde{\rho}_{\alpha} \quad (17)$$

Clearly,  $\tilde{\rho}_{\alpha}$  represents the mass of  $S_{\alpha}$  per unit volume of that constituent in the physical configuration. We assume  $\tilde{\rho}_{\alpha}$  remains finite so that  $\rho_{\alpha}$  will vanish whenever  $n_{\alpha}$  does. Next, we assume the constituents are isotropically arranged in the physical configuration, so that surfaces intersecting the composite will consist of constituent  $S_{\alpha}$  over  $n_{\alpha}$  of their area. Then we can define a crystal stress,  $\tilde{\mathbf{T}}_{\alpha}$ , for  $S_{\alpha}$  given by

$$\mathbf{T}_{\alpha} = n_{\alpha} \tilde{\mathbf{T}}_{\alpha} \quad (18)$$

Similarly, we have a crystal heat flux,  $\tilde{\mathbf{h}}_{\alpha}$ .

$$\mathbf{h}_{\alpha} = n_{\alpha} \tilde{\mathbf{h}}_{\alpha} \quad (19)$$

We may also introduce a crystal internal energy,  $\tilde{\epsilon}_{\alpha}$ . Since the internal energy is measured per unit mass of  $S_{\alpha}$  rather than per unit volume, we see that  $\epsilon_{\alpha}$  and  $\tilde{\epsilon}_{\alpha}$  will be identical, provided surface energy in the physical configuration is ignored. Finally, the mixture density,  $\rho$ , can be related to the constituent crystal densities,  $\tilde{\rho}_{\alpha}$ , by

$$\frac{1}{\rho} = \sum_{\alpha} \frac{c_{\alpha}}{\tilde{\rho}_{\alpha}} \quad (20)$$

as is easily proved by Equations (3) and (17).